A Study on Stability and Incipient Turbulence in Poiseuille and Plane-Poiseuille Flow by Numerical Finite-Difference Simulation

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A difference method for treating the complete two-dimensional Navier-Stokes equations in time-dependent form is applied to the prediction of incipient turbulence. A steady laminar flow profile is disturbed and the propagation of disturbances in time and space is calculated. Changes occur in amplitude and in character much as would be observed in a laboratory experiment. The classical paradox of stability of Poiseuille flow to low amplitude disturbances at all Reynolds numbers is studied and contrasted to plane-Poiseuille flow. The amplitude dependence of stability is demonstrated. The results are shown to be consistent with prior theoretical and experimental work. The work lends strong support to the difference approach to difficult stability problems.

The prediction of instability in Poiseuille flow is one of the most interesting unsolved classical problems in hydrodynamics. Aside from its historical interest and obvious practical value, there are interesting aspects of the problem that have caused many workers to study it over the years.

First, despite numerous valiant efforts starting with Sexls' (*14*) papers, all mathematical attempts to predict the observed hydrodynamic instability have failed. This failure is in contrast to the many successes of the theory in other flows. In particular, plane-Poiseuille flow has been analyzed successfully and the predicted critical Reynolds number is consistent with experimental observations. This contrast makes the apparent stability of Poiseuille flow all the more puzzling since the plane-Poiseuille flow shares the parabolic undisturbed velocity profile, and the two flows would appear to be much the same on superficial examination.

A second source of interest in the problem is that only in recent years has there been any sort of general agreement that the predicted stability was a result of a correct application of the linear theory. The linear theory reduces to a complicated eigenvalue problem which defies exact solution and which has taxed even the most able and diligent of workers. Hence, until recently many respected workers felt the predicted stability of Poiseuille flow to infinitesimal disturbances at all Reynolds numbers was in reality a result of an incomplete or deficient analysis, and that eventually a correct analysis would disclose a critical Reynolds number in agreement with experiment [see for example Birkhoff (*2*) and Lin (*12*)].

A third and the most interesting point is that several careful experiments indicate in a fairly conclusive way that Poiseuille flow is stable to all disturbances of sufficiently low amplitude. The classical linear theory of course deals with infinitesimal disturbances so that the findings of the theory are in agreement with experiment. The fact remains, however, that the observed instability of Poiseuille flow to "large" disturbances has never been predicted, and the response to infinitesimal disturbances apparently is entirely different than that of its near relative, plane-Poiseuille flow.

The literature is devoid of references to nonlinear studies on Poiseuille flow stability, which is perhaps not too surprising considering the difficulties encountered in years past in the linear problem. On the other hand, there have been some promising results on nonlinear plane-Poiseuille flow which will be discussed later.

The work to be presented here was undertaken with two objectives: First, it was intended to develop and assess a method for treating unstable laminar flows and the ensuing incipient turbulence by the use of numerical finite differences. Difference methods are well known for their power in numerous diverse applications in partial differential equations, but applications to hydrodynamic instability have received surprisingly little attention. Second, the study was intended to shed light on the intriguing Poiseuille-plane-Poiseuille instability problem as indicated above. This problem is of intrinsic interest and is ideal for critically testing results from the difference study since there are analytical results for the limiting, low amplitude case as well as some careful experimental measurements.

As will be outlined in detail below, these objectives were accomplished by direct finite-difference solution of the complete, two dimensional, Navier-Stokes equations. The solutions were carried out by starting with a steady laminar flow profile at specified Reynolds number. The profile was disturbed and the propagation of the disturbance in space and time was observed in much the same way as in a laboratory experiment. Very low amplitude disturbances were used to yield comparisons with the linear theory and to compare with the high amplitude results.

The use of a two-dimensional rather than a three-dimensional disturbance was based upon the experimental evidence of Leite (*11*), who demonstrated that for Poiseuille flow the unsymmetric part of a disturbance was more

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stable than the symmetric part of a disturbance, and the theoretical work of Squire (*16*), who proved for linear disturbances that two-dimensional disturbances are more stable than three-dimensional ones. Thus a two-dimensional disturbance will give a lower critical Reynolds number than three-dimensional disturbances, and of course it is clear that the computational difficulties of three dimensions are formidable.

The prior literature on analysis of stability with the linear theory is far too extensive to review here. Instead the reader is referred to the well-known monographs by Lin (*12*) and Chandrasekhar (3) and to recent papers by Gill (*10*) and Corcos and Sellars (*4*).

Nonlinear theoretical studies of stability have been confined almost exclusively to plane-Poiseuille flow. The work of Meksyn and Stuart (*13*) took into account the interaction between the disturbance and the mean flow, but not interactions between disturbances. They showed a distinct lowering of the minimum critical Reynolds number from 5,300 to 2,900 with increasing amplitude of the disturbance. Watson (*17*) formulated a problem which takes into partial account all interactions. The results indicate that a disturbance may amplify with distance to an equilibrium amplitude under certain conditions. That is, a stable laminar oscillation may develop which does not amplify or decay on the average with distance. Excellent summaries of nonlinear stability theory are given in a monograph by Eckhaus (7) and by Shen (*15*).

Experimental studies by Leite (*11*) have shown that Poiseuille flow is stable to small axisymmetric disturbances for Reynolds numbers up to 13,000 and that the critical Reynolds number decreases with increasing amplitude of the disturbance. Ekman has maintained laminar flow in a pipe up to Reynolds numbers of 40,000 by minimizing disturbances (8).

FORMULATION OF THE DIFFERENTIAL PROBLEMS

Navier-Stokes Equations

Consider the two-dimensional flow of an incompressible, isothermal fluid between two parallel plates or in a tube as shown in Figure 1. If we define the vorticity ω and the stream function ψ as

$$\omega = u_r - v_x;\; u = \psi_r, v = -\psi_x \quad \text{plane-Poiseuille} \quad (1)$$

$$\omega = r(u_r - v_x);\; u = (\psi_r)/r,\; v = (-\psi_x)/r \quad \text{Poiseuille} \quad (2)$$

where the subscripts denote differentiation. In both cases x denotes axial distance from the disturbance and r denotes distance from the center line in the usual way. We can eliminate pressure between the two components of the equation of motion and obtain the well-known vorticity transport equation. A further change of variables was made to increase the accuracy of the computation by considering the deviation from the steady laminar profile. Variables were defined as $\omega = g + \omega_o$; $\psi = \psi_o + f$; $u = u' + u_o$; and $v' = v$ where ω_o, ψ_o, and u_o refer to the steady state laminar flow solutions. It should be emphasized, that the equations were not linearized. The definitions of u' and v' are then

$$u' = f_r,\; v' = -f_x \quad \text{plane-Poiseuille} \quad (3)$$

$$u' = (f_r)/r;\; v' = (-f_x)/r \quad \text{Poiseuille} \quad (4)$$

The stream function equations are

$$g = f_{rr} + f_{xx} \quad \text{plane-Poiseuille} \quad (5)$$

$$g = f_{xx} + f_{rr} - (f_r)/r \quad \text{Poiseuille} \quad (6)$$

The vorticity transport equations are

$$g_t + ug_x + vg_r + v(\omega_o)_r = \frac{1}{N_{Re}}(g_{xx} + g_{rr}) \quad \text{plane-Poiseuille} \quad (7)$$

$$g_t + ug_x + vg_r - \frac{2gv}{r} = \frac{1}{N_{Re}}\{g_{xx} + g_{rr} - (g_r)/r\} \quad \text{Poiseuille} \quad (8)$$

Boundary Conditions

At the center line symmetry was assumed. The symmetry is implicit in the assumption of two-dimensional Poiseuille flow. At the wall the usual no-slip condition was imposed as indicated below.

$$\begin{aligned} f_x = f_r &= 0 \text{ on the solid boundary and} \\ f = g &= 0 \text{ at the center line} \end{aligned} \quad (9)$$

There is considerable freedom in the choice of initial conditions and the other boundary condition. An initial condition of a spatially decaying wave was selected. This condition permits direct comparison with prior work and results in a relatively rapid approach to a steady oscillation in stable flows. The condition was required to satisfy the boundary conditions and to be consistent with the disturbance at $x = 0$. The disturbance itself was selected to be periodic and to satisfy continuity as well as the boundary conditions. Equation (10), which defines f, is the expression both for the initial condition (evaluated where $t = 0$) and for the disturbance (evaluated at $x = 0$).

$$f = A_m f(r) \exp(-A_o x) \cos(A_r x) \cos(\beta t) \quad (10)$$

where $f(r)$ is a function which prescribes the radial shape of the disturbance. Several shapes were studied in detail, and it was concluded that the shape of the disturbance has some effect in an initial period; but of course the behavior of the system tends to become independent of the condition at large time and large x. Hence, the shape of the profile was selected with minimization of computer time in mind.

The relationships used for the disturbance profiles $f(r)$ are

$$f(r) = r - 2r^3 + r^5 \quad \text{plane Poiseuille} \quad (11)$$

$$f(r) = \tfrac{1}{2}r^2 - r^3 + \tfrac{1}{4}r^4 \quad \text{Poiseuille} \quad (12)$$

One of the most interesting problems concerns the selection of appropriate conditions for the downstream boundary. Ideally, the flow field extends an infinite distance so that for finite times the disturbance is always propagating into a region of an undisturbed laminar profile. With a finite computer, it is of course not possible to use this condition, and it is necessary to select an alternate condition which in some sense is similar. Three ways of dealing with this difficulty were considered. First, the condition could be imposed that the laminar profile be

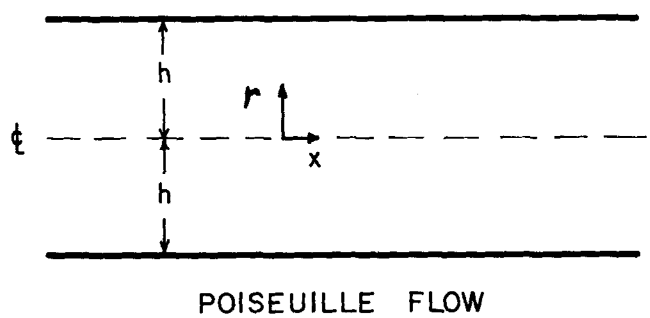

Fig. 1. Coordinates for plane-Poiseuille and Poiseuille flow.

established at a finite x corresponding to the length of the flow field selected for computation. Numerical experiments quickly established that this condition was unacceptable. The results of the calculation exhibited clear evidence of numerical instability starting from the downstream boundary.

Two acceptable methods for treating the condition were used:

$$f_x = -cf \tag{13}$$

$$f_{xx} = -A_r^2 f \tag{14}$$

In both cases it is easy to show that the function g must satisfy the same equation as the function f. Equation (13) corresponds to exponential spacial decay at the end of the flow field, whereas Equation (14) corresponds to a periodic continuative condition. Detailed experiments with various lengths of flow field revealed that either of these conditions was acceptable. The criteria for acceptability adopted were that the field length should have little or no influence on results at any fixed x position, and of course that there should be no evidence of numerical instability. Most of the calculations were done with Equation (14). This condition has previously been used in a study on an unconfined boundary-layer stability problem by DeSanto and Keller (5).

NUMERICAL METHOD

The literature on difference solution of problems involving hydrodynamic stability is surprisingly sparse. Several papers on prediction of incipient motion in natural convection have appeared. These cases, however, have been restricted to a much simpler situation which results in a stable laminar flow after a transition from an initial, motionless condition. The prior work in free convection has been reviewed recently (*1*) and hence will be omitted here. DeSanto and Keller (*5*) studied stability in an unconfined boundary-layer flow (Blasius flow) by a difference technique. They reported results of only two calculations with a relatively coarse grid used, so their results were not entirely conclusive. However, they were able to show that their approach definitely held promise and warranted further investigation. Their results were free of evidence of numerical instability and were consistent with prior work within the limitations of the model. Fromm and Harlow (*9*) have reported results on flow around blunt objects with an unstable wake using an explicit technique related to the Dufort-Frankel method for linear problems. Their work has stimulated considerable interest in the general area of computer simulation in hydrodynamics.

The basic numerical method used in this investigation has recently been described in detail elsewhere (*1*), so only an outline will be given here. Aziz and Hellums (*1*) have studied several methods in detail and have concluded that the one used in this work has important advantages in speed and/or accuracy over those previously reported. An alternating direction implicit method is used to advance the vorticity transport equation and successive optimum overrelaxation is used for the solution of the stream function equation. The method is nonlinear in that an iterative procedure is used at every time step to yield coefficients evaluated at the average time in the increment.

Some of the most interesting and difficult problems in studies of the type considered here are associated with the fact that it is entirely possible to compute an unstable "motion" which appears to be realistic, but which is in reality an artifact having no connection with fluid motion whatever. In other words under circumstances where the exact solution of the Navier-Stokes equations yields stable motion, an unstable finite-difference scheme can yield numerical results which have a completely different character. An important problem, then, is associated with distinguishing possible numerical instability from hydrodynamic instability. It is well known that theoretical work on the numerical stability problem cannot yield conclusive results in complicated problems such as those at hand. However, the techniques used are known to be stable under very general circumstances for the corresponding linear problem, and this knowledge leads one to expect stability in the nonlinear problem of interest. However, confirmation of numerical stability must come from direct digital computer experimentation.

An extensive series of numerical experiments was carried out for the purpose of establishing accuracy as well as numerical stability. Selected problems were reworked by using smaller and smaller space and time increments. The results of the studies indicated the methods used were strongly stable as well as accurate. The character of the flow did not change and the numerical values changed only slightly as the grid was refined. Details of these studies are reported elsewhere (*6*). Several runs were made by varying the length of the flow to establish that the artificial downstream boundary condition had little effect on the results. In most cases the flow field length was 37 h with $\Delta x = 0.175$, $\Delta y = 0.10$, and $\Delta t = 0.15$.

The computer time required for studies of this type is not excessive with efficient difference methods even on a computer which is relatively slow by modern standards. Each run must be continued until the major part of the transient effect of the initial condition has been swept downstream and the character of the flow is relatively time independent. Eight cycles of the disturbance cor-

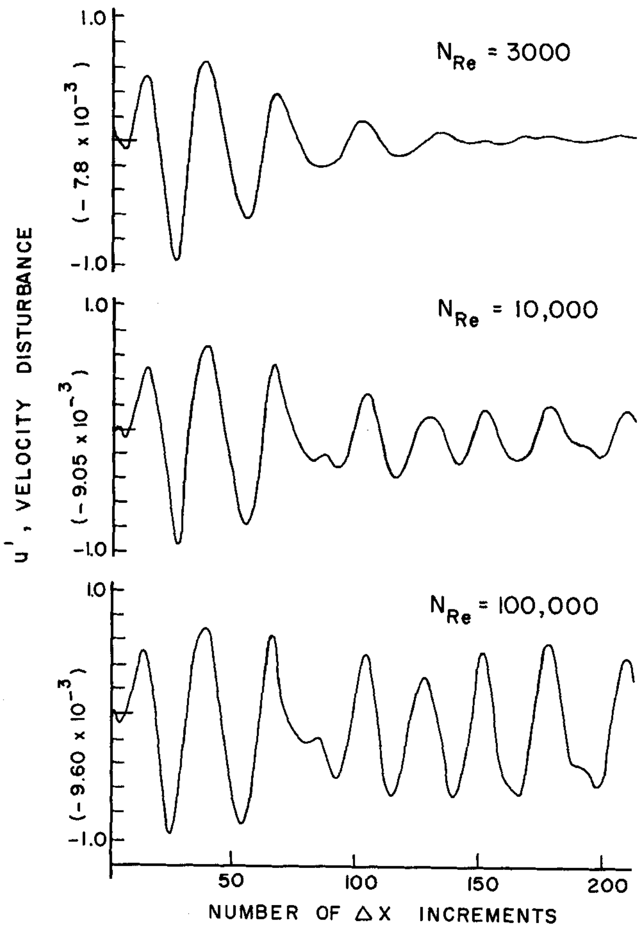

Fig. 2. Poiseuille flow response to high amplitude disturbance ($Am = 10^{-1}$, $r = 0.5$, $t = 48$).

responding to 320 time steps were found to be sufficient, and this required about 5 hr. on the Rice University computer for each run. The Rice University computer is about the same speed as the IBM 7040, although direct comparison is difficult.

RESULTS

Many parameters and functions must be specified to determine a problem of the type under consideration. The Reynolds number is of course the primary parameter. The parameters associated with the disturbance, Equation (10), are A_o, which determines the initial space decay rate; A_r, which determines initial space wavelength; and A_m the amplitude. After some experimentation $A_o = 0.1$ and $A_r = 1.0$ were chosen for all runs as being representative as nearly as possible of the behavior of the system after several cycles of the disturbance. These choices were intended to minimize the time of transient. The amplitudes considered were $A_m = 10^{-5}$ and $A_m = 10^{-1}$. The few available results on nonlinear analysis indicate that the results for $A_m = 10^{-5}$ should be comparable to those from linear theory and some computer experiments with $A_m = 0$ were made in confirmation. For $A_m = 10^{-1}$ nonlinear effects are definitely important. This amplitude yields fluctuations of 10% of the magnitude of the maximum undisturbed velocity. The frequency β was chosen to be unity in most cases. One series of tests was made in which β was varied over an interval designed to cross the point of neutral stability for plane Poiseuille flow.

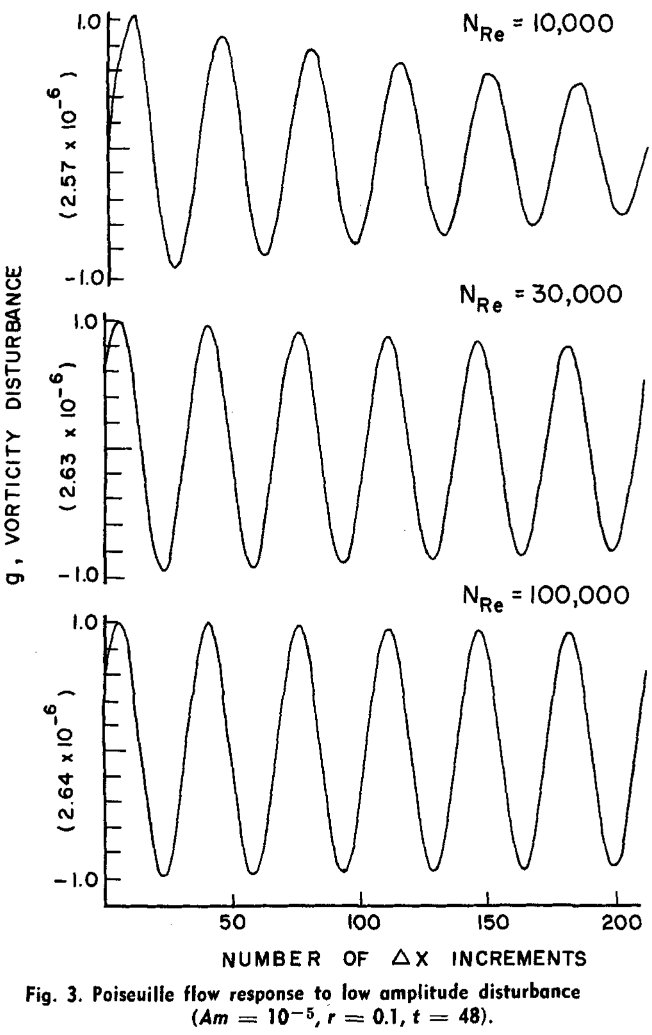

Fig. 3. Poiseuille flow response to low amplitude disturbance ($Am = 10^{-5}$, $r = 0.1$, $t = 48$).

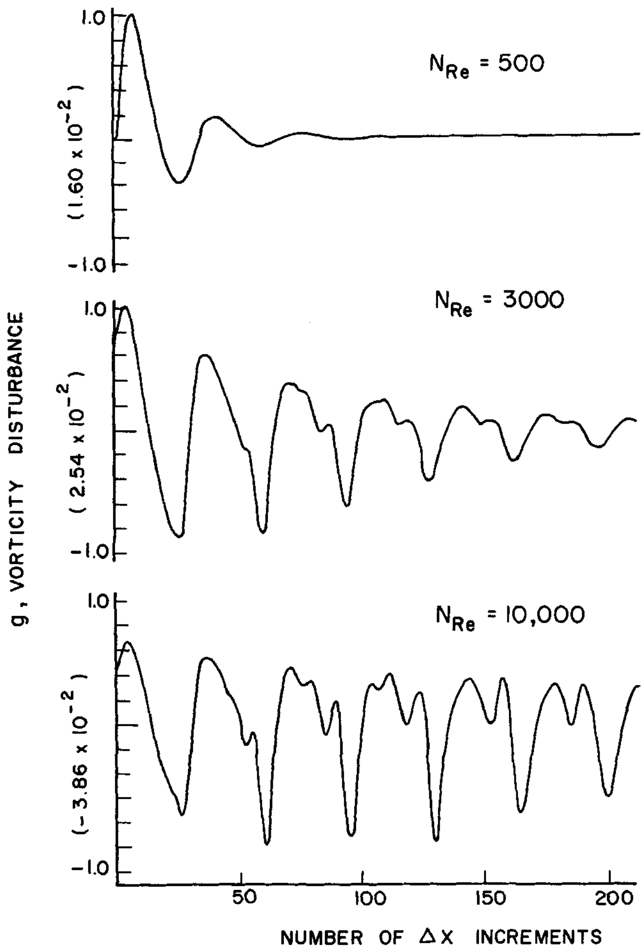

Fig. 4. Poiseuille flow response to high amplitude disturbance ($Am = 10^{-1}$, $r = 0.1$, $t = 48$).

A major difficulty in studies of this type is associated with the enormous volume of results produced, and the choice of methods for displaying and interpreting them. For the purposes of this presentation, selected results will be given in the form of graphs of an independent variable: f, g, or u' vs. x, the axial distance. This kind of presentation emphasizes the growth or decay of the disturbance with distance downstream, taken of course at a fixed radial position and at a fixed specified time, $t = 48$. The figures display the qualitative features of the flow in a direct way that is easy to visualize physically. The curves as presented are normalized for amplitude by division by the maximum magnitude of the function in the interval. Hence, the low amplitude results are "scaled" to appear to have the same amplitude in the figures as the high amplitude results. The maximum magnitude of the function is given in parenthesis on the ordinate of each figure.

A second important difficulty in presenting and interpreting the results is that clear evidence of stability or instability in some cases may require a longer flow field and a longer time than is feasible in a reasonable amount of computer time. In some runs, evidence of *instability* appears early at certain radial positions and slowly propagates elsewhere. Hence, one could draw completely contrary conclusions on looking at two different radial positions in an isolated way without surveying the entire flow field. *In addition, the choice of disturbance shape seems* to influence the early development as to radial position. This discussion is intended to emphasize the contrast with the linear theory in which simple yes or no answers are obtained, in principle, at least. In the nonlinear problem

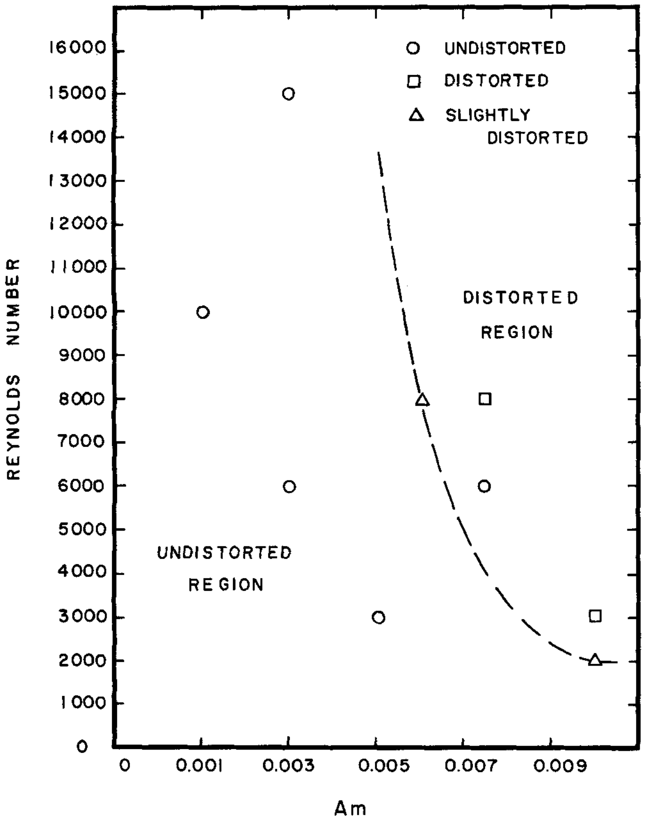

Fig. 5. Estimation of critical Reynolds number-amplitude relationship in Poiseuille flow.

at hand, as in the physical situation, the changes in the disturbance are more subtle, and are often localized to some extent when looking at a relatively short space-time domain of development. Despite these difficulties it will be shown that the incipient turbulence can be computed as it manifests itself in a clear change in the character of the flow.

Figure 2 displays the velocity fluctuation in Poiseuille flow in response to a high amplitude disturbance at three

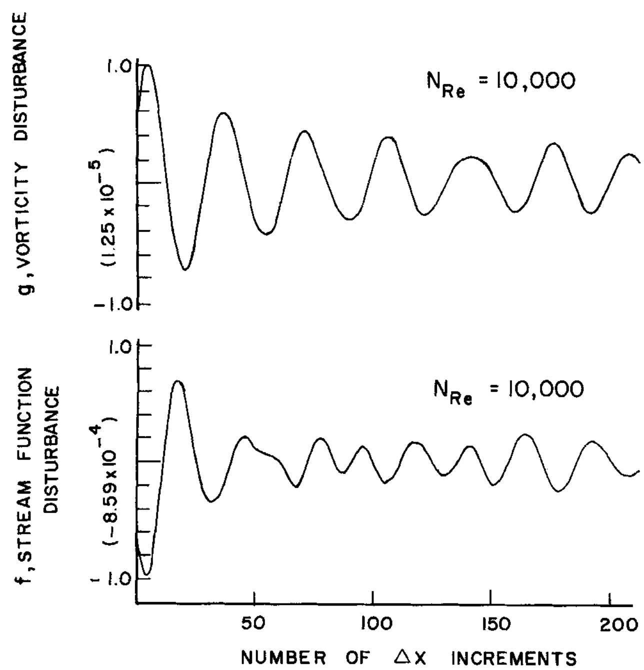

Fig. 6. Plane-Poiseuille flow response to low amplitude disturbance ($Am = 10^{-5}$, $r = 0.1$, $t = 48$).

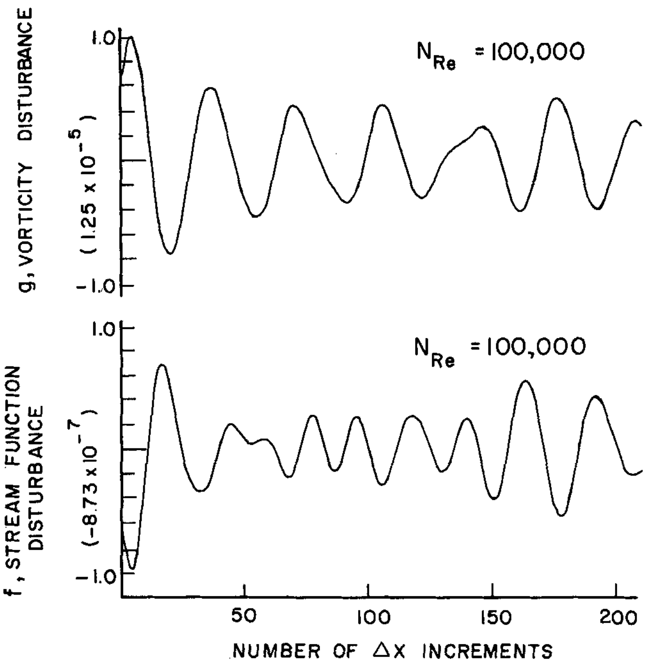

Fig. 7. Plane-Poiseuille flow response to low amplitude disturbance ($Am = 10^{-5}$, $r = 0.1$, $t = 48$).

different Reynolds numbers. Lower Reynolds numbers results, not shown, exhibit a rapid decay similar to the Reynolds number of 3,000. Notice that the flow at $N_{Re} = 3{,}000$ shows little evidence of instability. The wave is slightly distorted from the sinusoid, but seems to be decaying with distance. Presumably, at some greater space-time position the disturbance would exhibit a reamplification. However, this effect is not apparent at Reynolds numbers very near the critical in the restricted flow field of the present work.

The results in Figure 2 for $N_{Re} = 10{,}000$ and 100,000 clearly exhibit the character of instability. At $N_{Re} = 10{,}000$ the disturbance decays slightly at small values of x and then exhibits reamplification and distortions from the sinusoid with no evidence of tendency to decay. These effects are qualitatively the same at $N_{Re} = 100{,}000$ with a stronger tendency toward amplification.

The results in Figure 2 were presented in terms of the velocity fluctuation, which is the dependent variable most easily measured and visualized. It is instructive also to display values of the vorticity fluctuation. The vorticity seems to be a more sensitive indicator of instability than velocity in some cases. Specifically, comparison of low and high amplitude results in terms of vorticity fluctuations shows changes in character of the flow which are much more pronounced than changes in the same results expressed in terms of velocity fluctuations. Figures 3 and 4 show the amplitude dependence of stability of Poiseuille flow. The low amplitude disturbance in Figure 3 is shown to be monotone decaying in amplitude at all Reynolds numbers with no evidence of distortion of the wave form. In contrast, Figure 4 presents results of the identical case except with a high amplitude disturbance. At $N_{Re} = 500$ the disturbance decays rapidly as expected, whereas at $N_{Re} = 3{,}000$ the first evidence of instability is apparent in the distorted wave form, although there is no reamplification in the flow field used. Finally at $N_{Re} = 10{,}000$ the instability is clear in marked contrast to the same results for a low amplitude disturbance.

A series of runs was made on the amplitude-Reynolds number relationship with a flow field of 8 h and a slightly different disturbance profile. Use of this short flow field

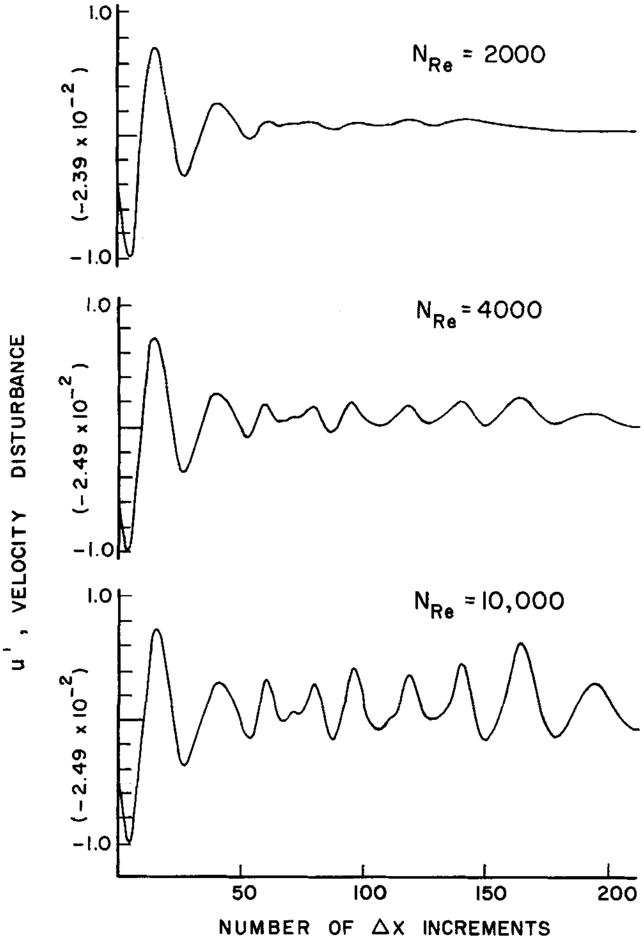

Fig. 8. Plane-Poiseuille flow response to high amplitude disturbance ($Am = 10^{-1}$, $r = 0.5$, $t = 48$).

made it possible to make a number of runs without prohibitive computer time requirement, although of course the detection of subtle changes in the flow is less accurate in the short field. Nevertheless, it was found that the appearance of distortions in wave form of the vorticity fluctuation are reproducible and give a reasonably well-defined Reynolds number-amplitude relationship, as shown in Figure 5. The results show both the strong amplitude dependence of stability and a minimum critical Reynolds number of about 2,000 as expected. A relationship as precise as is desired presumably could be established by extending the time, the flow field, and the number of cases computed.

The contrast in behavior of Poiseuille flow with that of plane-Poiseuille flow is evident from comparison of Figures 3 and 4 with Figures 6 and 7. Figures 6 and 7 give results for plane-Poiseuille flow with a low amplitude disturbance. It can be seen that the results bear little similarity to the comparable Poiseuille flow results. Rather the disturbance is clearly not decaying and bears much more resemblance to the high amplitude Poiseuille flow. Hence, the amplitude dependence of the character of Poiseuille flow is seen to be much stronger than the comparable case of plane-Poiseuille flow in agreement with prior work. It should be pointed out that at some radial positions in Poiseuille flow there was evidence of instability to low amplitude disturbances. It cannot be said with assurance that these disturbances would decay given a sufficiently long flow field. Hence, while this work clearly shows the much stronger amplitude dependence of Poiseuille flow than plane-Poiseuille flow, it cannot be regarded as conclusive evidence of the complete stability of Poiseuille flow to infinitesimal disturbances.

Figures 6 and 7 give the stream function fluctuation in addition to the vorticity fluctuation to indicate the relationships between the dependent variables. The variables display the same character with only minor differences in most cases. The velocity and stream function fluctuations are especially closely related.

Plane-Poiseuille flow with a high amplitude disturbance is displayed in Figure 8 in a way suitable for comparison with Figure 2 on Poiseuille flow. The flows respond in a very similar way to high amplitude disturbances in agreement with experimental work. Comparisons with the corresponding low amplitude results in Figures 6 and 7 show that the amplitude affects a change from a nearly sinuous motion to distorted one. These results may be regarded as evidence in support of a stable oscillation as predicted by Watson (*17*).

A number of other cases have been studied and the results are available in detail (*6*). One study which produced only negative results was made on the frequency dependence of plane-Poiseuille flow. The dimensionless frequency was varied over the range 0.75 to 1.25 which brackets the curve of neutral stability predicted from the linear theory at $N_{Re} = 10{,}000$. The changes in the character of the flow were insufficient to form the basis for any conclusions. These findings need not be contrary to the linear theory, however, since the frequencies were near neutral stability. Conceivably a much longer flow field could reveal a clear difference in the character of the flows.

CONCLUSION

The results of this study show conclusively that numerical finite-difference simulation can be of great value in complicated hydrodynamic stability problems. The computer time requirement is moderate, and difficulties associated with numerical instability are absent. The Reynolds numbers and disturbance-amplitude dependence of the character of both Poiseuille and plane-Poiseuille flow have been studied. Apparently there has been no successful prior work of any kind on the amplitude effect in Poiseuille flow.

Other comments in summary are:

1. The results for Poiseuille flow are consistent with the linear theory which predicts stability to small disturbances at all Reynolds numbers. In contrast, plane-Poiseuille flow exhibits a critical Reynolds number for disturbances of all amplitudes.

2. Disturbance amplitude is shown to have a very pronounced effect in Poiseuille flow. Sufficient calculations were done to permit determination of the approximate shape of the neutral stability curve.

3. The results for plane-Poiseuille flow are consistent with prior experimental and theoretical work. The results seem to support the stable oscillation predicted by Watson.

ACKNOWLEDGMENT

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NOTATION

A_m = amplitude constant as defined by Equation (10)
A_o = initial decay rate constant as defined by Equation (10)
A_r = initial spacial frequency constant as defined by Equation (10)

c = constant
f = deviation in stream function from the undisturbed laminar profile
$f(r)$ = function describing wave shape of disturbance in Equation (10)
g = deviation in vorticity from the undisturbed laminar profile
h = radius of conduit or half the plate spacing
N_{Re} = Reynolds number $U^o h/\nu$
r = dimensionless radial coordinate
t = dimensionless time
u, v = dimensionless velocity components in x, y directions, respectively
u' = deviation in u from undisturbed laminar profile
U^o = maximum (center line) velocity of the steady laminar flow
x = dimensionless coordinate downstream from disturbance
β = dimensionless disturbance frequency as defined by Equation (10)
ψ = dimensionless stream function
ω = dimensionless vorticity
ν = kinematic viscosity

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Some Observations on the Effect of Interfacial Vibration on Saturated Boiling Heat Transfer

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An experimental investigation was carried out to determine the effects of vibration of the heat transfer surface in saturated pool boiling of water at atmospheric pressure. Wires of 0.01 in. diameter were heated electrically and vibrated electromagnetically at frequencies ranging from 20 to 115 cycles/sec. and amplitudes from 0.0118 to 0.0701 in. An increase in heat transfer up to a maximum of 200% at low ΔT was observed for an increase in frequency and/or amplitude. At a heat flux of 10^5 B.t.u./(hr.)(sq. ft.) high-speed motion pictures were taken at 4,800 frames/sec. of a wire vibrating at 45 cycles/sec. with an amplitude of 0.0492 in. Comparison of these films with those taken at the same heat flux without vibration showed that the generating period and diameters at break-off for the pulsed wire follow normal distribution. The waiting period is much longer and more fluctuating in nature. A slight increase in bubble emission frequency was also observed for pulsating wire.

Experimental and theoretical investigations in the field of nucleate boiling heat transfer have indicated that the flow of heat is partly from the surface to the liquid through the boundary layer and partly by latent heat transported through the vapor bubbles. However, photographic studies on subcooled boiling by Gunther and Kreith (*1*), Rosenhow and Clark (*2*), and others have proved that heat transfer takes place primarily through the boundary layer, the transport by latent heat being negligible. As the large heat fluxes obtained cannot be explained on the basis of thermal conduction through the superheated layer, it has been postulated that certain random microconvection is created in the boundary layer by bubble dynamics, reducing thus the high thermal resist-